

# Crystal Structure Prediction with Improved Initialization

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Russian  
Science  
Foundation

# Crystal Structure Prediction from First-principles

Structure prediction drives materials discovery

Artem R. Oganov<sup>1,2,3\*</sup>, Chris J. Pickard<sup>4,5\*</sup>, Qiang Zhu<sup>6</sup> and Richard J. Needs<sup>7</sup>

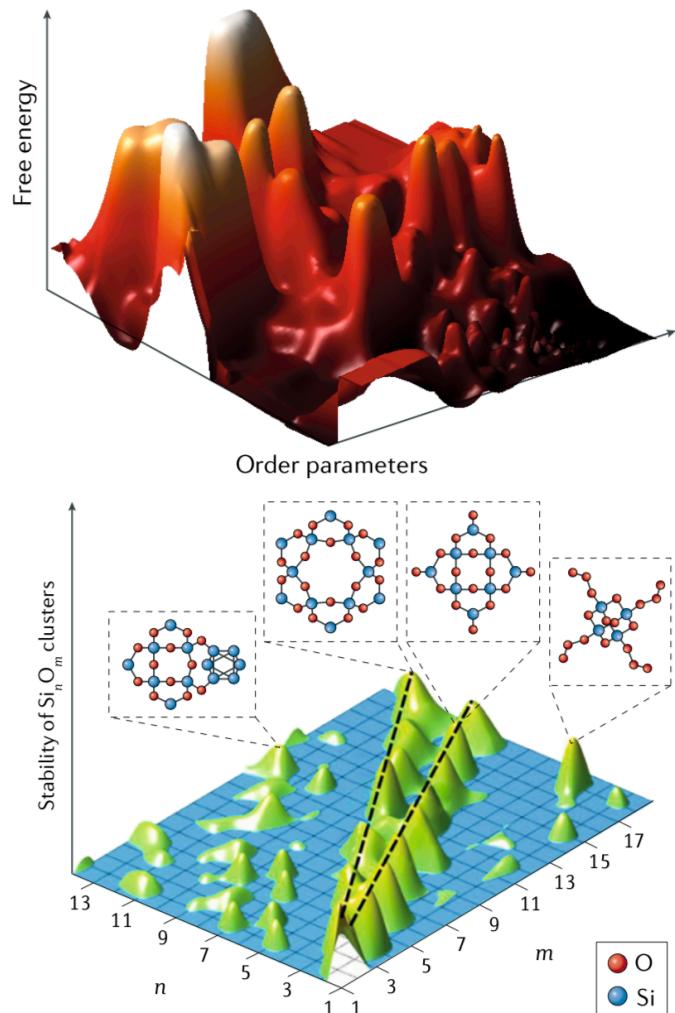
*Nature Review Materials*, 2019

## - Structure navigation

- Evolutionary algorithm (USPEX)
- Random search (AIRSS)
- Minima hopping
- Some others .....

## - Energy & Properties Evaluation

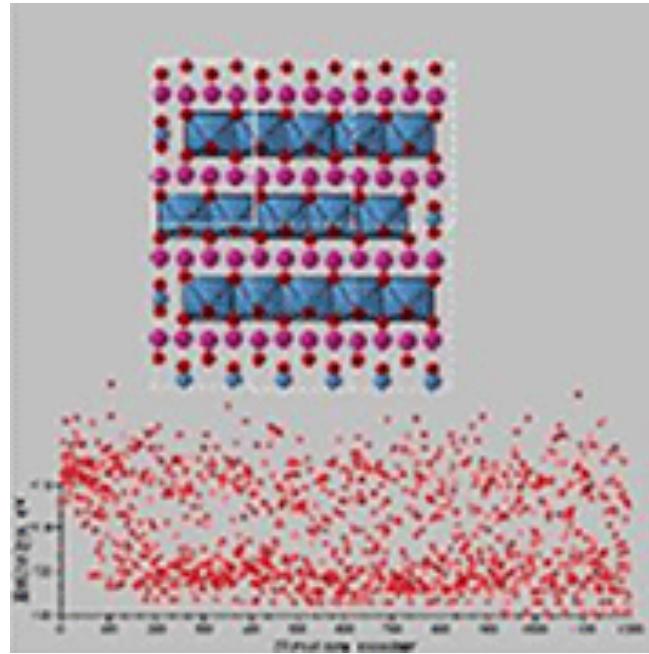
- Evolutionary algorithm (USPEX)
- Random search (AIRSS)
- Minima hopping
- Some others .....



# Universal Structure Predictor: Evolutionary Xtallography

Inorganic crystals

Oganov, JCP, 2006

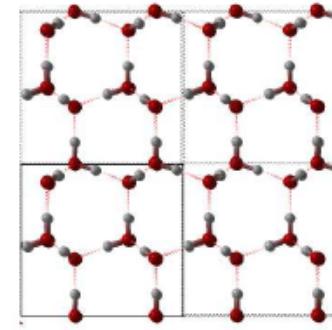


$\text{MgSiO}_3$

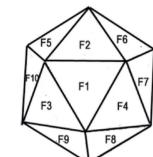
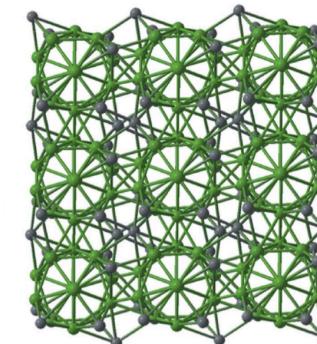
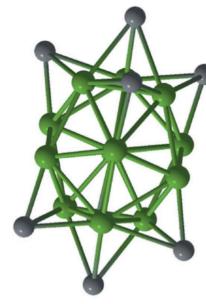
Organic crystals

Zhu, Acta Cryst B, 2012

$\text{H}_2\text{O}$



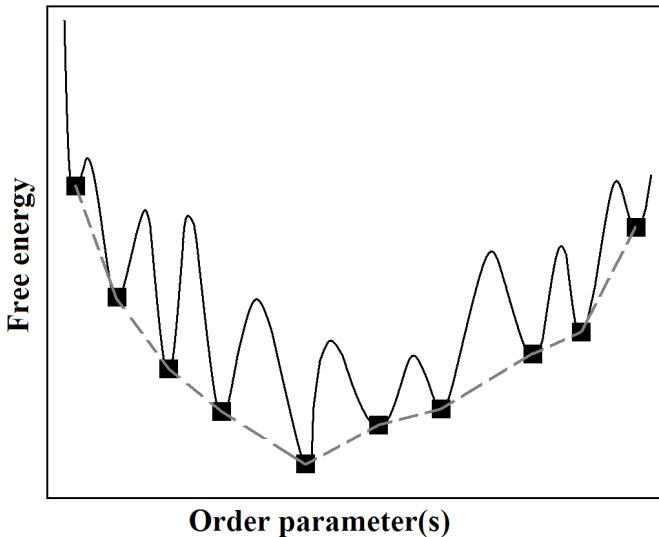
Methane A



# CSP Strategy

## Global optimization

## Structural Navigation *derivative-free*



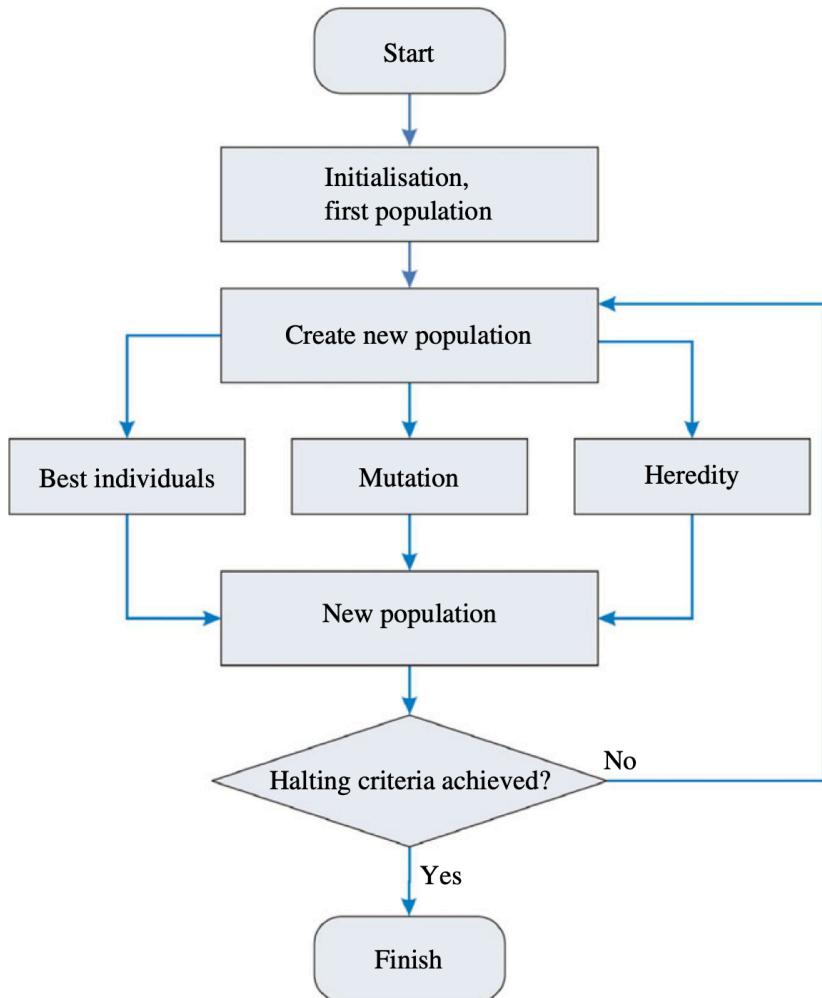
## Local optimization

## Geometry Optimization *Convex optimization*

- (Quasi)-random search
- (Quantum) simulated annealing
- Minima/Basin hopping
- Bio-inspired algorithms
  - Evolutionary/genetic
  - Particles swarm
  - Ant colony
  - Firefly

- DFT
- Semi-classical
- Generic (customized) force field
- Machine learning

# CSP Strategy (Population based methods)



All population based methods require an initialization

- Purely random
- Random symmetrization
- Random topology (Bushlanov, CPC, 2019)

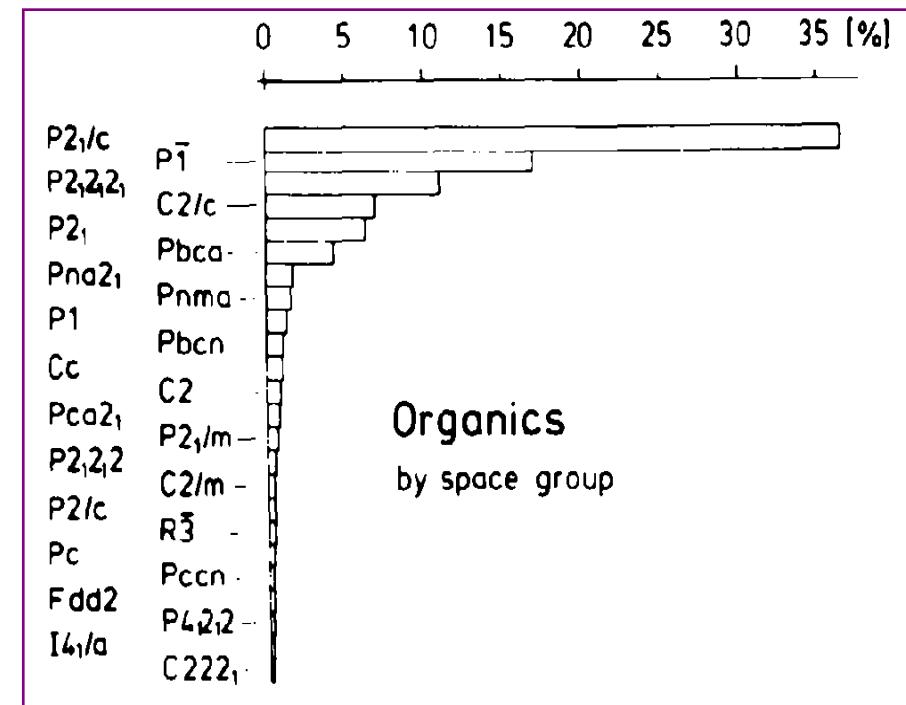
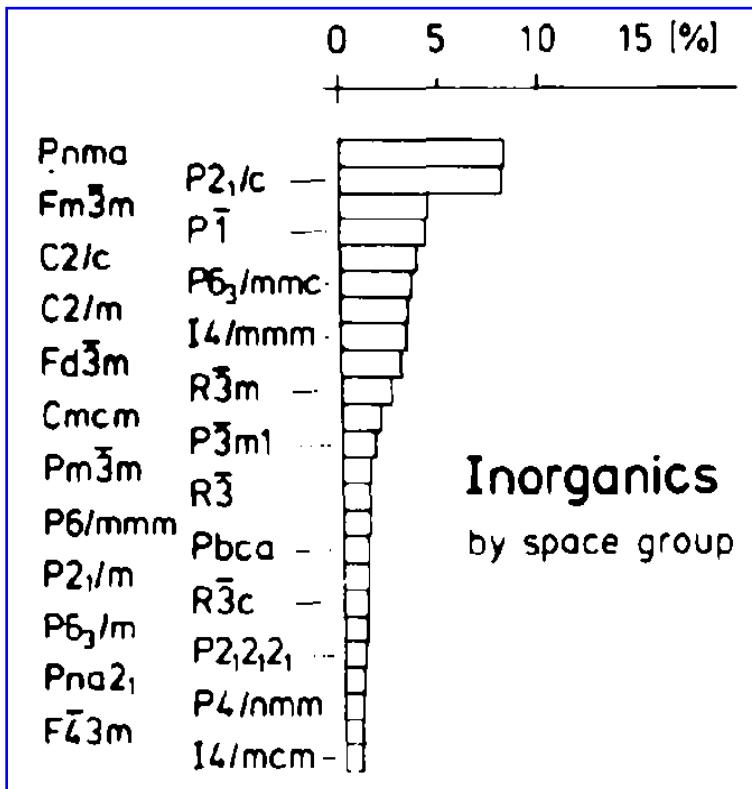
In this talk, I will focus on the random symmetrization based on the open source code PyXtal.



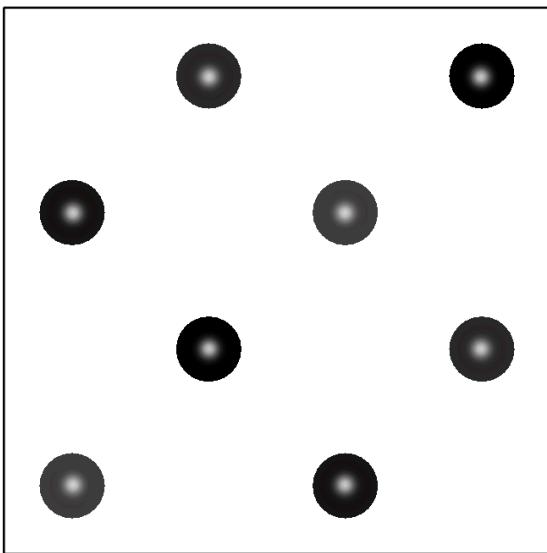
Fig. 1. Flowchart of a typical evolutionary algorithm.

Lyakhov, CPC, 2013

# Symmetry Preference



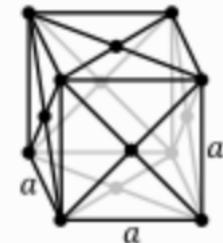
# How to describe a crystal by symmetry?



A diamond structure

Space group: Fd-3m (227)

Bravi lattice: Face-centered cubic



Atomic Positions

|      |     |     |
|------|-----|-----|
| 1/8  | 1/8 | 1/8 |
| 7/8  | 3/8 | 3/8 |
| 1/8  | 5/8 | 5/8 |
| 7/8  | 7/8 | 7/8 |
| 5/8  | 1/8 | 5/8 |
| 11/8 | 3/8 | 7/8 |
| 5/8  | 5/8 | 1/8 |
| 11/8 | 7/8 | 3/8 |

Wyckoff Sites

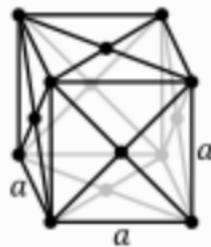
|      |
|------|
| 192i |
| 96h  |
| 96g  |
| 48f  |
| 32e  |
| 16d  |
| 16c  |
| 8b   |
| 8a   |



# How to randomly generate a diamond-like crystal?

Space group: Fd-3m (227)  
8 carbon atoms

1. Build a FCC lattice



2. Choose Wyckoff sites

Wyckoff Sites

|      |
|------|
| 192i |
| 96h  |
| 96g  |
| 48f  |
| 32e  |
| 16d  |
| 16c  |
| 8b   |
| 8a   |

3. Fill atomic positions

Atomic Positions

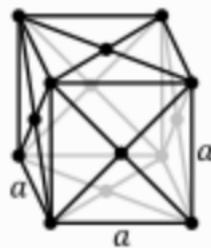
|      |     |     |
|------|-----|-----|
| 1/8  | 1/8 | 1/8 |
| 7/8  | 3/8 | 3/8 |
| 1/8  | 5/8 | 5/8 |
| 7/8  | 7/8 | 7/8 |
| 5/8  | 1/8 | 5/8 |
| 11/8 | 3/8 | 7/8 |
| 5/8  | 5/8 | 1/8 |
| 11/8 | 7/8 | 3/8 |

# How to randomly generate a diamond-like crystal?

Space group: Fd-3m (227)

**16** carbon atoms

1. Build a FCC lattice



2. Choose Wyckoff sites

Wyckoff Sites

|      |
|------|
| 192i |
| 96h  |
| 96g  |
| 48f  |
| 32e  |
| 16d  |
| 16c  |
| 8b   |
| 8a   |

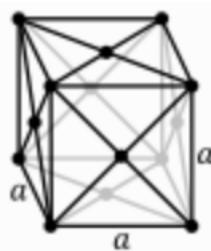
3. Fill atomic positions

- 16c
- 16d
- 8a + 8b

# How to randomly generate a diamond-like crystal?

Space group: Fd-3m (227)  
**32** carbon atoms

1. Build a FCC lattice



2. Choose Wyckoff sites

Wyckoff Sites

|      |
|------|
| 192i |
| 96h  |
| 96g  |
| 48f  |
| 32e  |
| 16d  |
| 16c  |
| 8b   |
| 8a   |

3. Fill atomic positions

- 32e
- 16c + 16d
- 8a + 8b + 16c
- 8a + 8b + 16d

32e

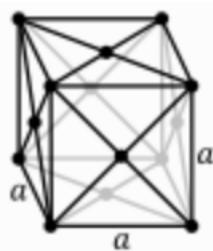
|     |     |     |
|-----|-----|-----|
| x   | x   | x   |
| -x  | -x  | -x  |
| ... | ... | ... |

**High multiplicities  
means more freedoms**

# How to randomly generate a diamond-like crystal?

Space group: Fd-3m (227)  
**32** carbon atoms

1. Build a FCC lattice



2. Choose Wyckoff sites

Wyckoff Sites

|      |
|------|
| 192i |
| 96h  |
| 96g  |
| 48f  |
| 32e  |
| 16d  |
| 16c  |
| 8b   |
| 8a   |

3. Fill atomic positions

- 32e
- 16c + 16d
- 8a + 8b + 16c
- 8a + 8b + 16d

192i

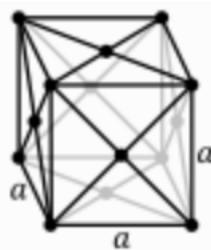
| x   | y   | z   |
|-----|-----|-----|
| -x  | -y  | -z  |
| ... | ... | ... |

**High multiplicities  
means more freedoms**

# How to randomly generate a diamond-like crystal?

Space group: Fd-3m (227)  
**32** carbon atoms

1. Build a FCC lattice



2. Choose Wyckoff sites

Wyckoff Sites

|      |
|------|
| 192i |
| 96h  |
| 96g  |
| 48f  |
| 32e  |
| 16d  |
| 16c  |
| 8b   |
| 8a   |

3. Fill atomic positions

- 32e
- 16c + 16d
- 8a + 8b + 16c
- 8a + 8b + 16d

192i

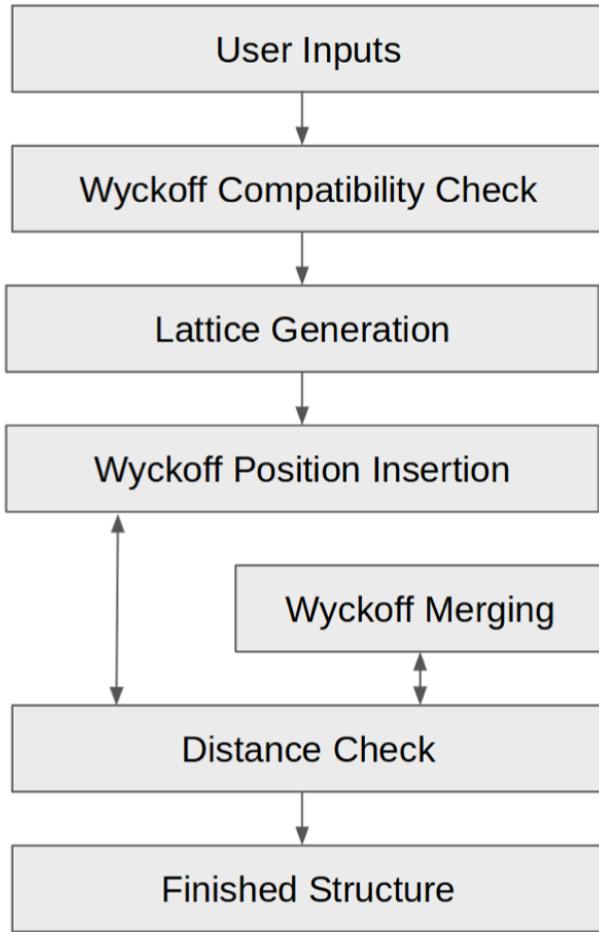
| x   | y   | z   |
|-----|-----|-----|
| -x  | -y  | -z  |
| ... | ... | ... |

*How to choose the right combination of Wyckoff sites?*

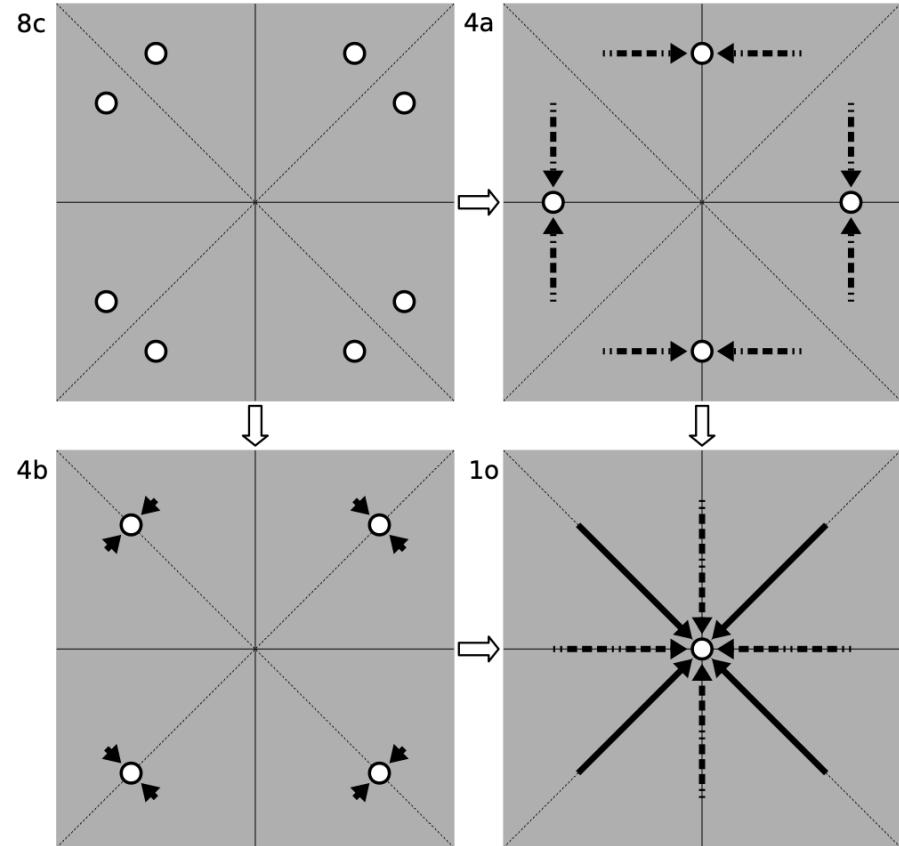
- Precompute all the allowed solution, then randomly pick one
- **Start with the highest possible sites, then merge it (to avoid selecting too high-symmetrical results)**

**High multiplicities  
means more freedoms**

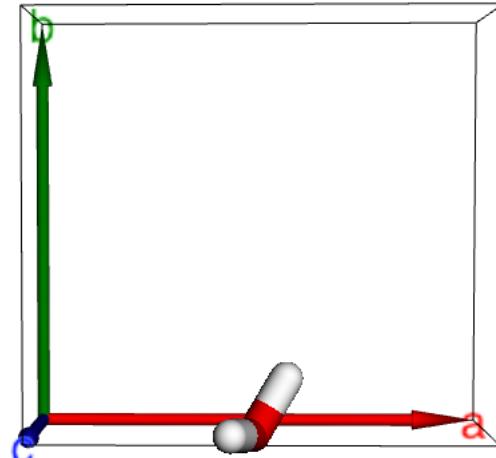
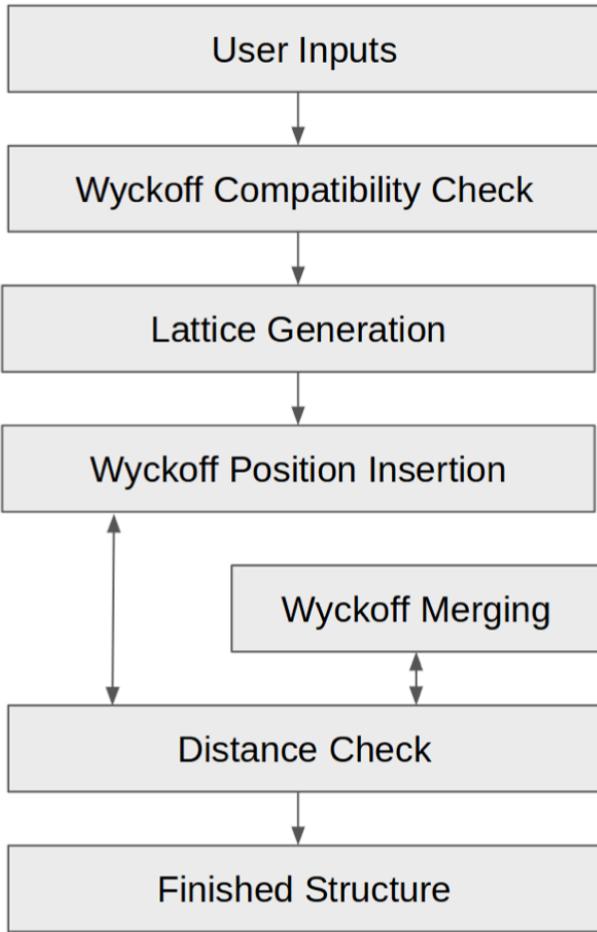
# Wyckoff Site Selection & Merging



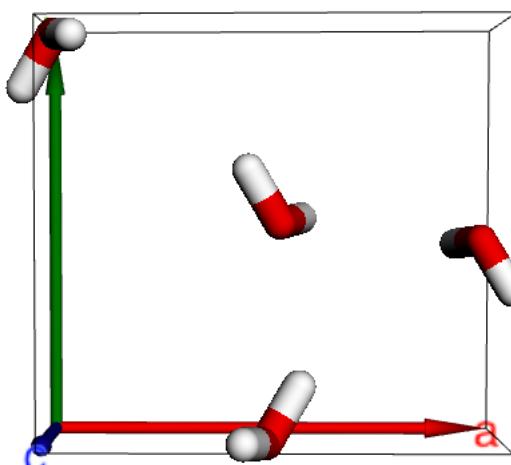
Space group:  $C_{4v}$  4mm



# How to generate a molecular crystal?



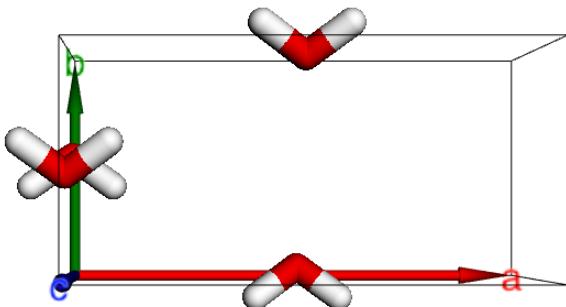
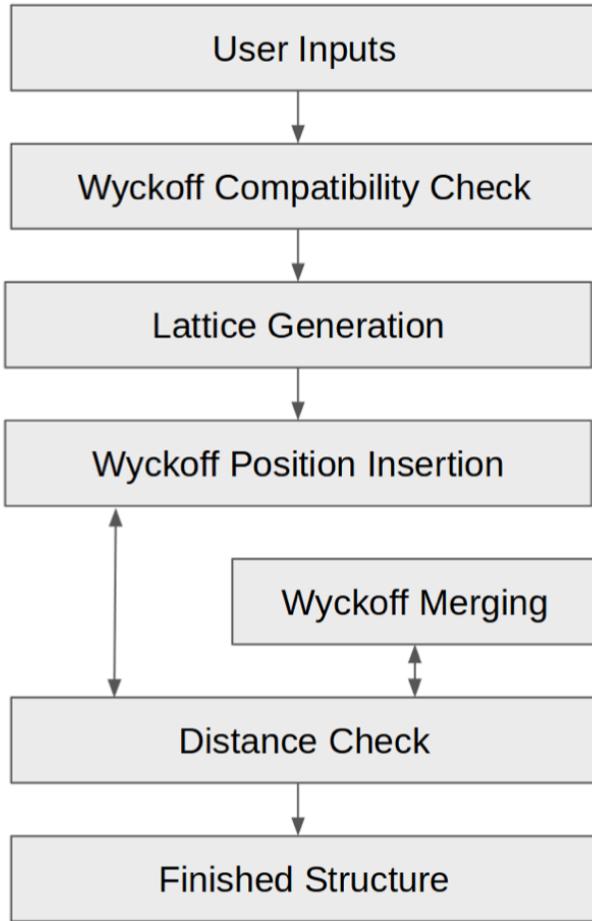
Space group:  $P2_12_12_1$   
# of molecules: 4  $\text{H}_2\text{O}$   
Cell params: optional  
Wyckoff sites: optional



| Site | Symmetry |
|------|----------|
| 4a   | m..      |

| x        | y        | z        |
|----------|----------|----------|
| $-x+1/2$ | $-y$     | $z+1/2$  |
| $-x$     | $y+1/2$  | $-z+1/2$ |
| $x+1/2$  | $-y+1/2$ | $-z$     |

# How to generate a molecular crystal with $Z' < 1$ ?



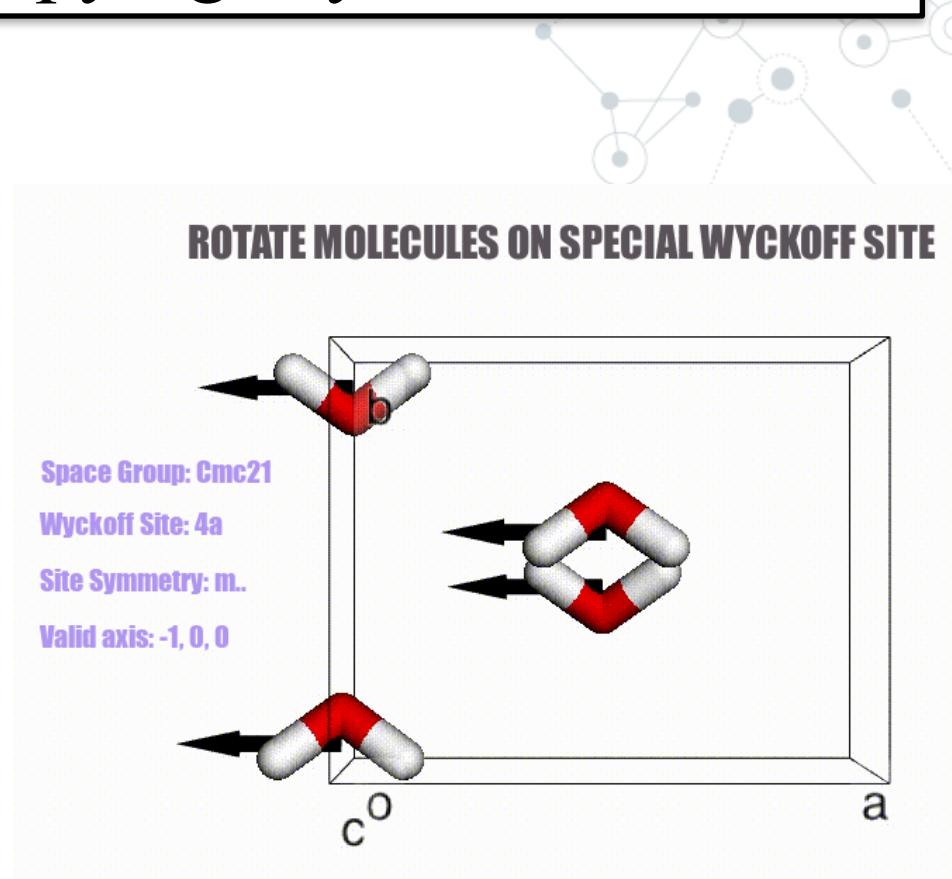
Space group:  $Cmc2_1$   
# of molecules: 4  $\text{H}_2\text{O}$   
Cell params: optional  
Wyckoff sites: optional

| Site | Symmetry |
|------|----------|
| 8b   | 1        |
| 4a   | m..      |

|       |          |         |
|-------|----------|---------|
| 0     | y        | z       |
| 0     | -y       | $z+1/2$ |
| $1/2$ | $y+1/2$  | z       |
| $1/2$ | $-y+1/2$ | $z+1/2$ |

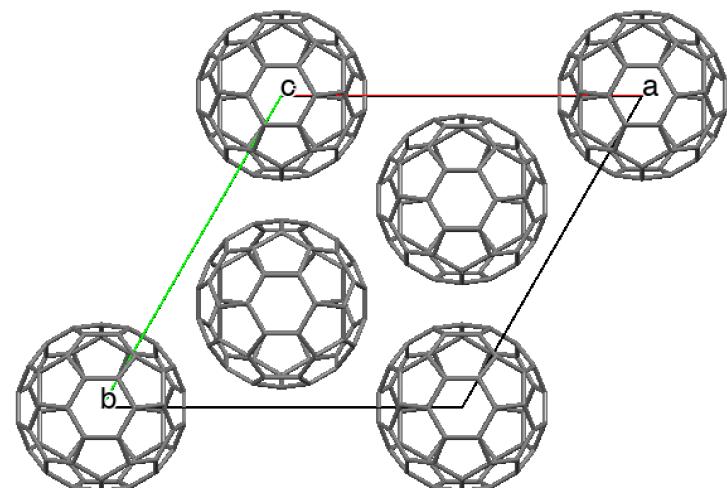
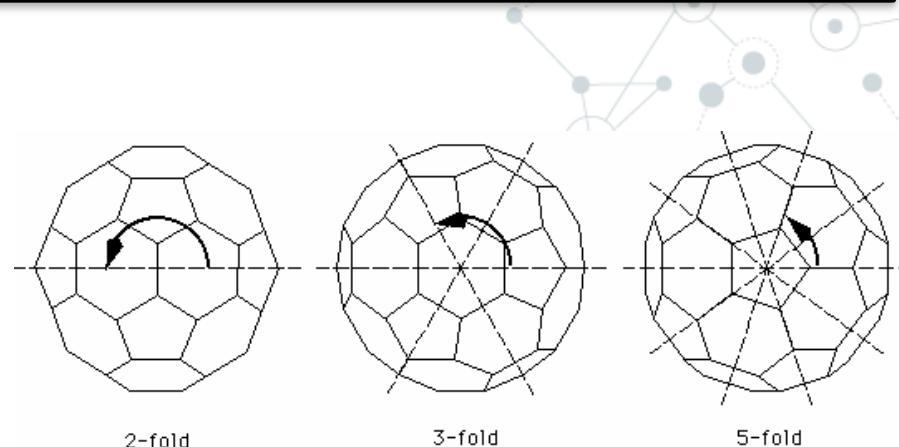
# Molecular structures occupying Wyckoff Positions

- (Special) Wyckoff positions have site symmetry
- Molecular symmetry must be equal or greater
- Orientation must be correct
- All valid orientations will be calculated by PyXtal
- $Z'$  can be any integer and fractional numbers



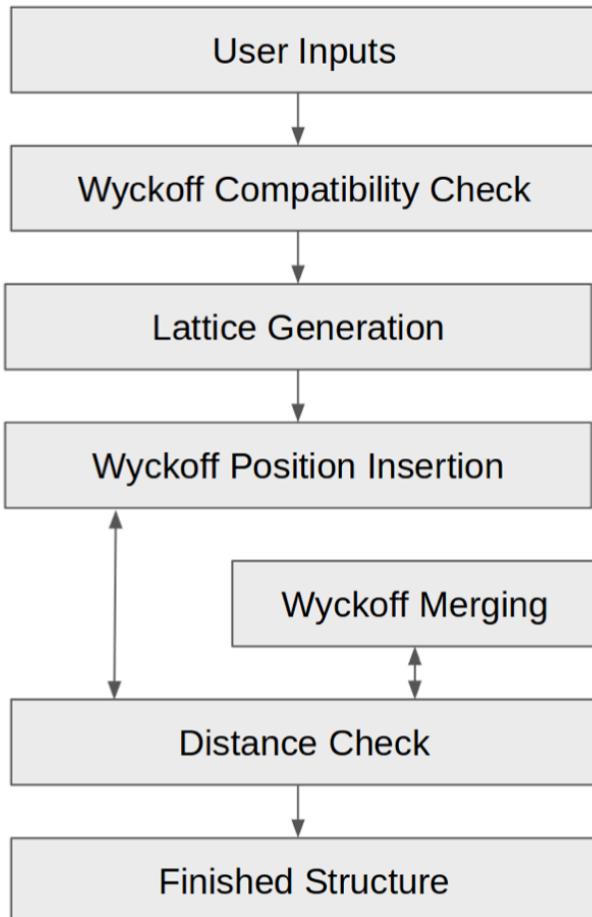
# Molecular structures occupying Wyckoff Positions

- (Special) Wyckoff positions have site symmetry
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- Orientation must be correct
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- $Z'$  can be any integer and fractional numbers

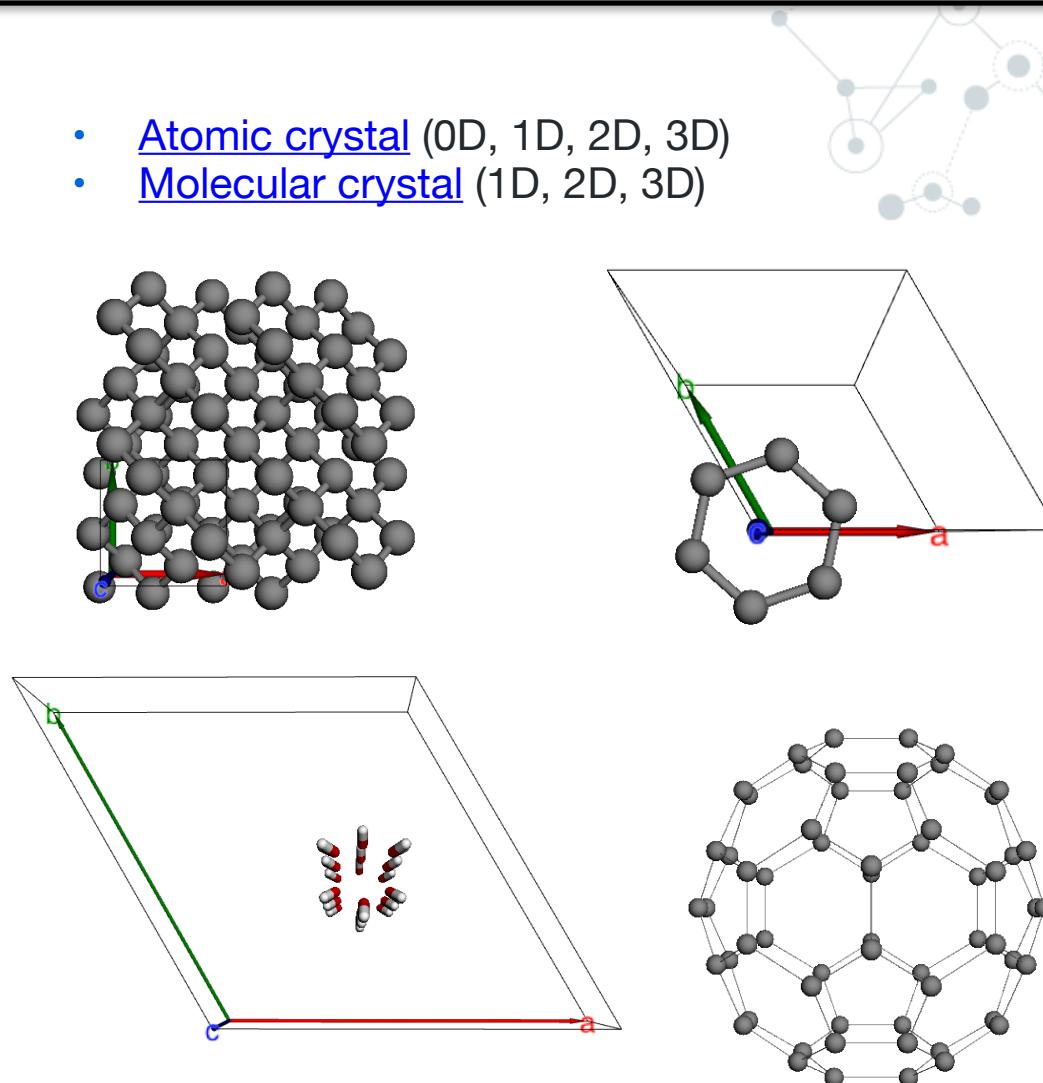


C<sub>60</sub>: P6<sub>3</sub>mc ( $Z=4$ ,  $Z'=1/3$ )

# More examples on symmetric generation



- [Atomic crystal](#) (0D, 1D, 2D, 3D)
- [Molecular crystal](#) (1D, 2D, 3D)



<https://pyxtal.readthedocs.io>

# Vary the crystals: Symmetry invariant

```
from pyxtal.molecular_crystal import molecular_crystal  
  
h2o = molecular_crystal(36, ['H2O'], [2])  
print(h2o)
```

-----Random Molecular Crystal-----

Dimension: 3

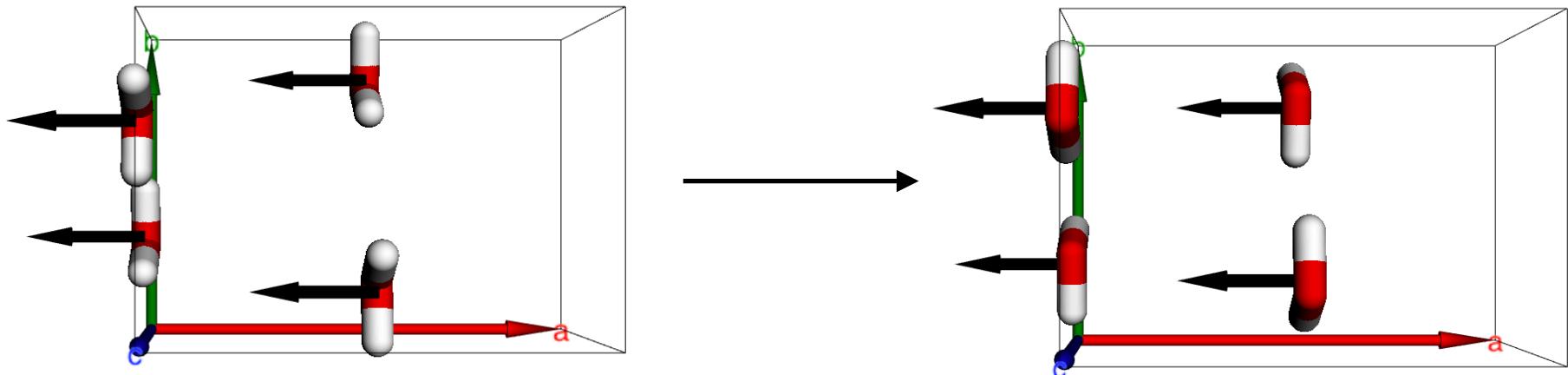
Group: Cmc21

Volume factor: 1.1

orthorhombic lattice: 4.9933 5.6751 6.1478 90.0000 90.0000 90.0000

Wyckoff sites:

H2O1 @ [0.0000 0.4828 0.4315] WP: 4a, Site symmetry m.. ==> Euler: -172.390 -47.970 -108.176



Rotation & translation according to the given constraints

# Vary the crystals: Lower the symmetry

```
1 from pyxtal.crystal import random_crystal  
1 C = random_crystal(227, ['C'], [2], 1.0)  
1 print(C)
```

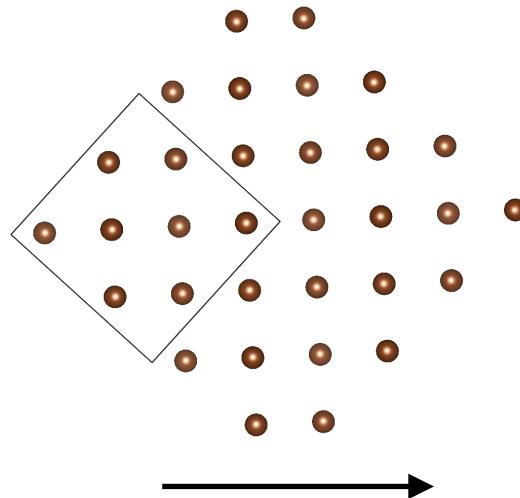
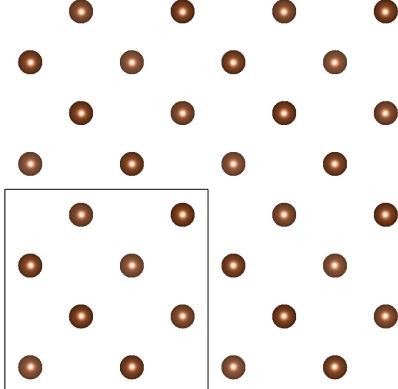
```
-----Random Crystal-----  
Composition: C8  
Dimension: 3  
Group: Fd-3m (227)  
Volume factor: 1.0  
cubic lattice: 5.1495 5.1495 5.1495 90.0000 90.0000 90.0000  
Wyckoff sites:  
    C @ [0.1250 0.1250 0.1250], WP: 8a, Site symmetry: -4 3 m
```

Supported since PyXtal-v0.1.2

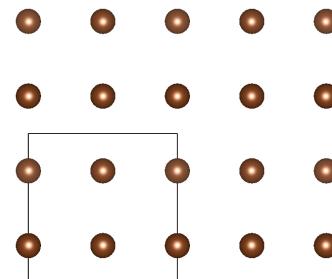
```
1 C1 = C.subgroup(H=141)  
2 print(C1)
```

```
-----Random Crystal-----  
Composition: C8  
Dimension: 3  
Group: I41/amd (141)  
Volume factor: 1.0  
tetragonal lattice: 3.6413 3.6413 5.1495 90.0000 90.0000 90.0000  
Wyckoff sites:  
    C @ [0.0200 0.7718 0.1204], WP: 4a, Site symmetry: -4 m 2
```

Fd-3m  
8a



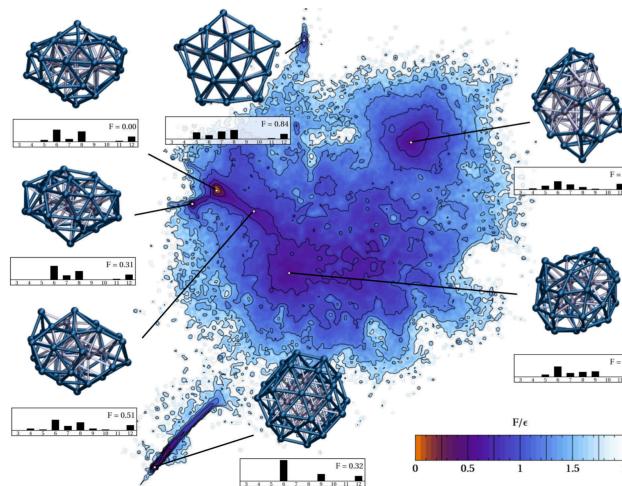
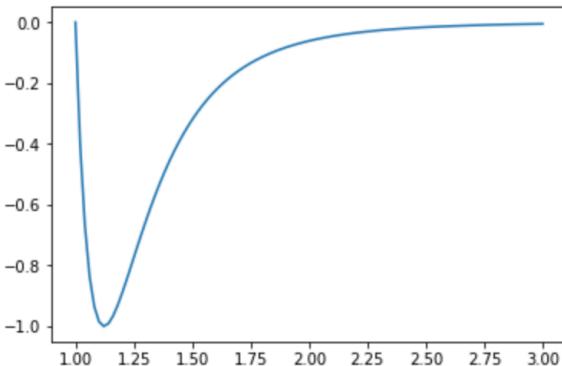
I41/amd  
4a



Transformation matrix  
Wyckoff splitting scheme

# Structure Optimization: Lennard-Jones Clusters

$$V(r) = 4\epsilon \left[ \left( \frac{\delta}{r} \right)^{12} - \left( \frac{\delta}{r} \right)^6 \right]$$

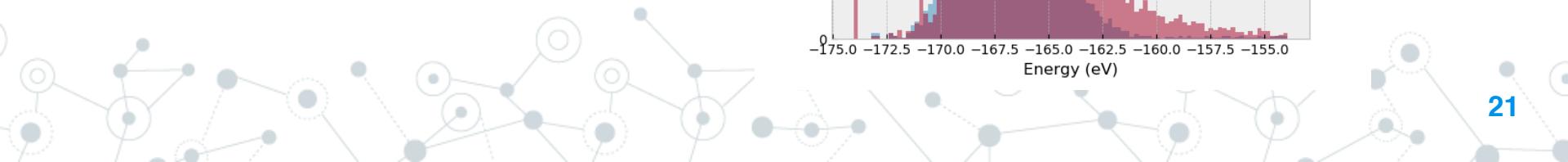
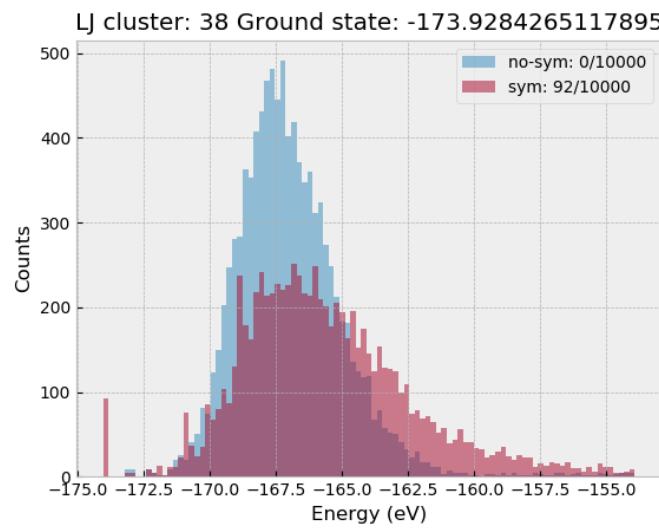


Ceriotti, Tribello, Parrinello, JCTC, 2013

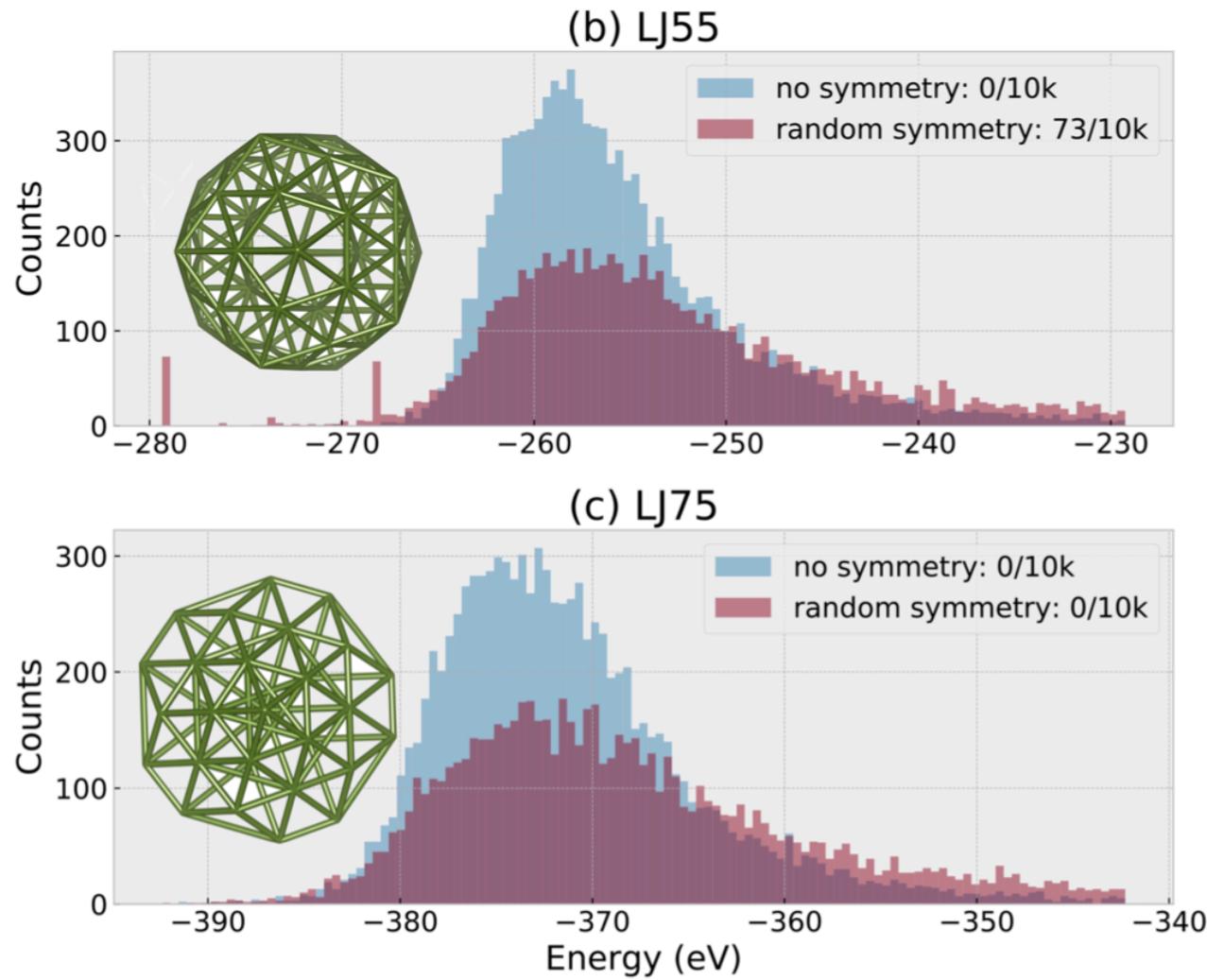
## Random sampling



- 10000 random structures
- then optimize



# Structure Optimization: Lennard-Jones Clusters

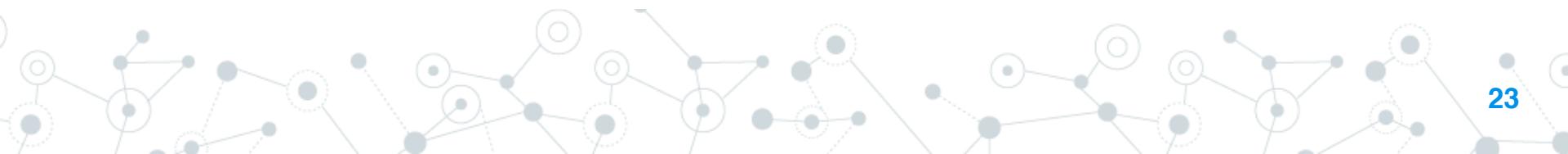
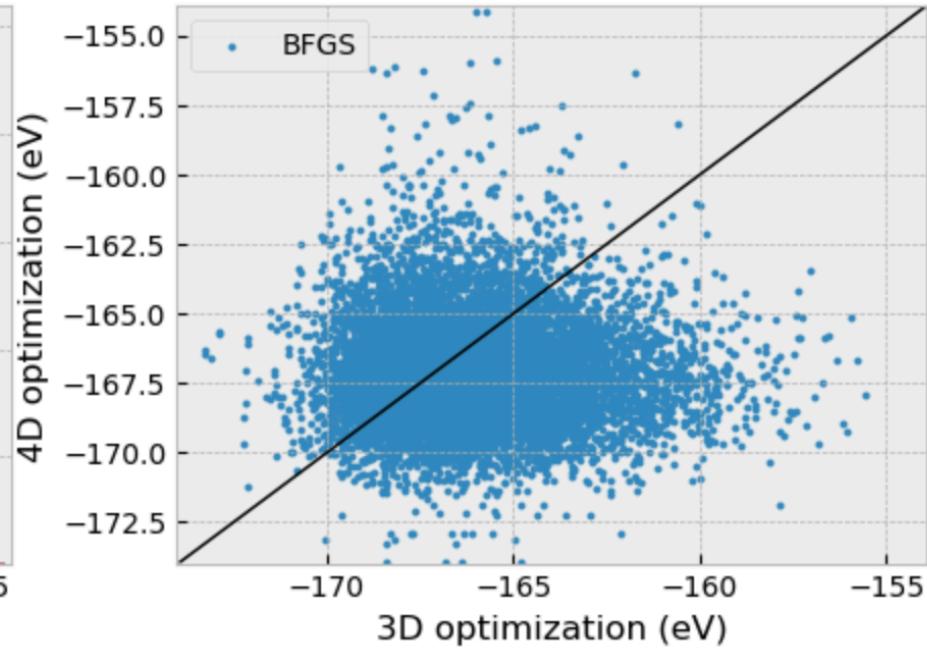
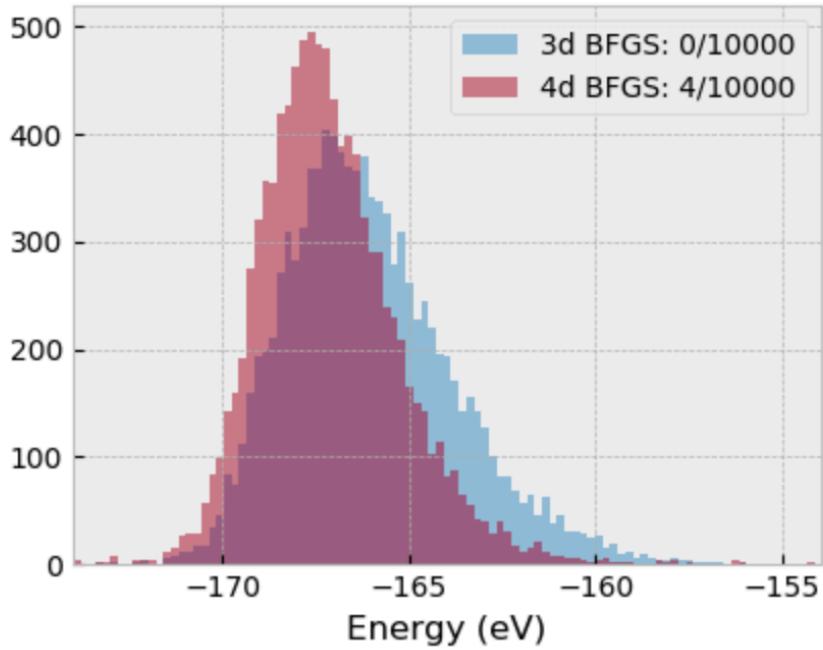


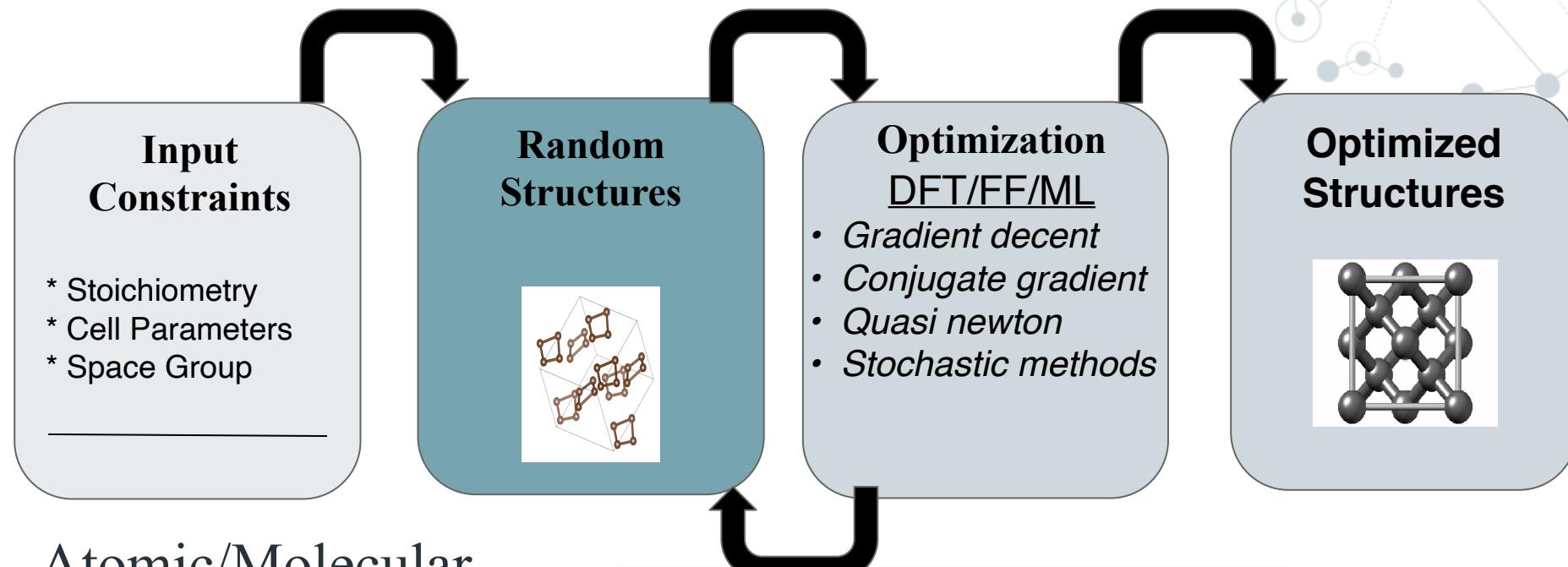
Structure prediction can be improved by handling structure generation.

# Structure Optimization: Exploratory algorithms

- 10000 random structures with  $C1$  symmetry
- Optimize them differently
  - Ordinary L-BFGS
  - Stochastic hyperspatial optimization

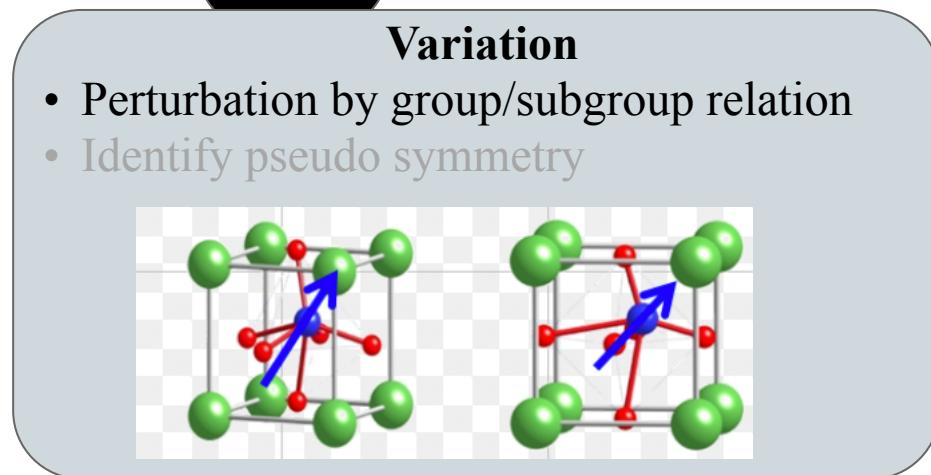
*C. J. Pickard, PRB, 2019*





## Atomic/Molecular

- 4D: superspace group
- 3D: space group
- 2D: Layer group
- 1D: Rod group
- 0D: Point group



# Summary

**PyXtal** is an open source code to deal with structures

## Generation

- Generation of atomic/molecular crystals from 0D, 1D, 2D to 3D
- Crystals with molecules in special Wyckoff positions ( $Z' < 1$ )

## Modulation

- Group-subgroup relation
- Interface with Pymatgen/ASE for structural manipulation
- Exploratory optimization algorithms

## Analysis

- Symmetry information
- PXRD



2018/04

Launched by Scott

Supervised by Zhu



2019/07

Dee-y created the logo

Scott graduated

Dean/Stanley  
implemented XRD



2019/09

Available in pip install

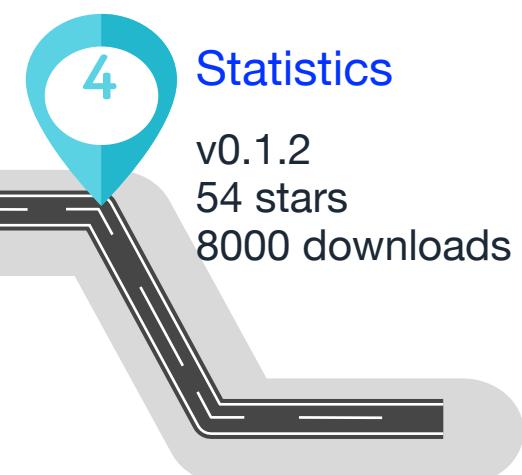
Submitted the paper



2020/11

Visualization

Group-subgroup function



## Statistics

v0.1.2  
54 stars  
8000 downloads

## Contributors

Scott Fridericks

Dean Sayre

Stanley Baronett

Kelvin Parrish

Qiang Zhu

Dee-y

Phys Goodall

Janosh Riebesell



## Outside

## How to contribute?

- pip install pyxtal
- Star our project in Github
- Suggestions
- Fork the repository
- Send your pull request

# Acknowledgement

Merci  
谢谢  
Спасибо  
**Grazie**  
Thank You  
ありがとう



## Skoltech

- A. R. Oganov

## UNLV

- Scott Fredericks
- Howard Yanxon
- David Zagaceta
- Kevin Parish

### 1-2 postdocs opening in

- Molecular simulation
- Code development
- Forcefield development

Feel free to contact [qiang.zhu@unlv.edu](mailto:qiang.zhu@unlv.edu)

